is not universal but depends on the relation which is being generalized. Ness' assumed expression for F is independent of the variation of θ_{tr} along the surface (since θ_{tr} is constant in the similarity case). On the other hand, the constant b in the Thwaites method [see Eq. (8) of Ness' reply] does depend on $d\theta_{tr}/d\beta$ (as can readily be shown) and, therefore, in the nonsimilar case, a variation of θ_{tr} along the surface is accounted for by Thwaites since β varies. Thus, Ness' method fails because he has generalized on a relation valid for similarity, which does not contain parameters that can be important in nonsimilar flow, such as $d\theta_{tr}/d\beta$.

4. With regard to Ness' example to show that his method "gives good results, at least, for incompressible flow," we repeat: What is essential to us is that any method which claims to be general must include all effects that can be shown to be important in limiting cases. It is impressive that his method checks the Blasius calculation so well, but it still errs by a large factor in the case discussed in paragraph 2.

In conclusion, it appears inherent in the Ness method that it can never properly predict the limiting behavior of a flow that is developing toward a terminal similarity different from its initial similar behavior. This conclusion does not apply if the modification proposed by the writers[2] is included. What Ness has not made clear is: When, except for the Blasius calculation cited, is his original method the more accurate?

References

[1] Ness, N., "Some comments on the laminar compressible boundary-layer analysis with arbitrary pressure gradient," AIAA J. **5**, 330–331 (1967).

[2] Ohrenberger, J. T. and Cohen, C. B., "Comments on a proposed variation of the Cohen-Reshotko method in boundary layer theory," AIAA J. **5**, 383–384 (1967).

Comments on "Effects of Energy Dissipation on a Spinning Satellite"

Y. PIRONNEAU*

University of Nantes, Nantes, France

THE author's approach[1] to this difficult problem is very interesting. However, one wonders if it is possible to affirm as they make their conclusion: "So far as stability is concerned, one may thus expect to have considerable latitude in choosing the size of the inner body."

In order to study the stability, they conserve only a part of the solution of Eqs. (7) and (8). When we take into account the neglected part, Eqs. (3–6) become differential equations with the coefficients function of the variable τ. A more complete study would show that despite this the system remains stable.

Evidently, the amplitude of the neglected terms tends towards zero exponentially, and consequently we could say that after a sufficient length of time their influence is negligible. However, it is possible that at this moment some of the angles determining the orientation of the satellite take values sufficiently large that it is no longer possible to replace the equations of motion by their linear forms [Eqs. (3–6)].

It is possible to obtain the complete solution of Eqs. (7) and (8). These equations can be written in the following form:

$$MX'' + BX' + KX = 0$$

with

$$X = \begin{vmatrix} \theta_3 \\ \psi_3 \end{vmatrix} \qquad M = \begin{vmatrix} (K+1) & 0 \\ 0 & K'+1Q \end{vmatrix}$$

$$B = \Delta Q \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} \qquad K = pQ \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix}$$

The form of the matrices M, B, and K shows that it is possible to diagonalize them. The eigenvalues are the roots of the characteristic equation,

$$\begin{bmatrix} [1-(K+1)\lambda] & -1 \\ -1 & [1-(K'+1)Q\lambda] \end{bmatrix} = (K+1)(K'+1)Q\lambda^2 - [(K+1) + (K'+1)Q]\lambda = 0$$

$$\lambda_1 = 0 \qquad \text{and} \qquad \lambda_2 = \frac{(K+1) + (K'+1)Q}{(K+1)(K'+1)}$$

The corresponding eigenvectors are, respectively,

$$Z_1 = \begin{vmatrix} 1 \\ 1 \end{vmatrix} \qquad \text{and} \qquad Z_2 = \begin{vmatrix} (K'+1)Q \\ -(K+1) \end{vmatrix}$$

The matrix of modes is

$$Z = \begin{vmatrix} 1 & (K'+1)Q \\ 1 & -(K+1) \end{vmatrix}$$

We can calculate

$$\tilde{Z}MZ = \begin{vmatrix} [(K+1)+(K'+1)Q] & 0 \\ 0 & (K+1)(K'+1)Q[(K+1)+(K'+1)Q] \end{vmatrix}$$

$$\tilde{Z}BZ = \Delta Q[(K+1)+(K'+1)Q]^2 \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix}$$

$$\tilde{Z}KZ = pQ[(K+1)+(K'+1)Q]^2 \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix}$$

Changing the variables

$$\begin{vmatrix} \theta_3 \\ \psi_3 \end{vmatrix} = Z \begin{vmatrix} q_1 \\ q_2 \end{vmatrix}$$

leads to the equations

$$\begin{cases} q_1'' = 0 \\ (K+1)(K'+1)q_2'' + \Delta[(K+1) + (K'+1)Q]q_2' + p[(K+1)+(K'+1)Q]q_2 = 0 \end{cases}$$

the solution of which is (the index zero indicating the value when $\tau = 0$)

$$\begin{cases} q_1 = q_{10} + q_{10}'\tau \\ q_2 = e^{-\lambda\tau}\left[q_{20}\cos\omega_2\tau + \dfrac{q_{20}' + \lambda q_{20}}{\omega_2}\sin\omega_2\tau\right] \end{cases}$$

with

$$\omega_2 = \omega_0\cdot(1-\epsilon^2)^{1/2}$$

$$\omega_0^2 = p\,\frac{(K+1)+(K'+1)Q}{(K+1)(K'+1)} = \frac{k}{\Omega^2}\,\frac{I_3+J_3}{I_3J_3}$$

$$\epsilon = \frac{\Delta}{2}\left[\frac{(K+1)+(K'+1)Q}{p(K+1)(K'+1)}\right]^{1/2} = \frac{\delta}{2}\left(\frac{I_3+J_3}{kI_3J_3}\right)^{1/2}$$

$$\lambda = \epsilon\omega_0 = (\delta/2\Omega)\cdot[(I_3+J_3)/I_3J_3]$$

$$\begin{vmatrix} q_{10} \\ q_{20} \end{vmatrix} = Z^{-1}\begin{vmatrix} \theta_{30} \\ \psi_{30} \end{vmatrix} \qquad \begin{vmatrix} q_{10}' \\ q_{20}' \end{vmatrix} = Z^{-1}\begin{vmatrix} \theta_{30}' \\ \psi_{30}' \end{vmatrix}$$

$$Z^{-1} = \frac{1}{I_3+J_3}\begin{vmatrix} I_3 & J_3 \\ I_1 & -I_1 \end{vmatrix}$$

Received February 9, 1967.

* Professor of Mechanics and Director, Ecole Nationale Supérieure de Mécanique.

and, finally

$$\begin{vmatrix}\theta_3 = q_1 + (K' + 1)Qq_2 = q_1 + (J_3/I_1)q_2 \\ \psi_3 = q_1 - (K + 1)q_2 = q_1 - (I_3/I_1)q_2\end{vmatrix}$$

The amplitude of q_2 decreases as $e^{-\lambda\tau} = e^{-\lambda\Omega \cdot t}$. The damping ratio ϵ possibly being very weak, nothing indicates a priori that we can neglect q_2 in relation to q_1. Unfortunately, keeping the exact values of θ_3 and ψ_3, the study of the stability must be much more complicated.

Reference

[1] Kane, T. R. and Barba, P. M., "Effects of energy dissipation on a spinning satellite," AIAA J. 4, 1391–1394 (1966).

Reply by Authors to Y. Pironneau

T. R. KANE* AND P. M. BARBA†
Stanford University, Stanford, Calif.

PIRONNEAU'S elegant solution of Eqs. (7) and (8) can be used to show that both $\theta_3' - \alpha$ and $\psi_3' - \alpha$ can be made arbitrarily small for all τ by a suitable choice of initial values of θ_3, ψ_3, θ_3', and ψ_3'. It is precisely this fact that justifies use of Eqs. (9) in conjunction with Eqs. (3–6) to study the stability problem posed in the paper, because only θ_3' and ψ_3' (but not θ_3 and ψ_3) appear in Eqs. (3–6). [This also applies to the full, nonlinear equations from which Eqs. (3–6) were derived.] In other words, our analysis does, in fact, take the complete solutions of Eqs. (7) and (8) into account.

Received March 6, 1967.
* Professor of Engineering Mechanics. Member AIAA.
† Graduate Student; presently Senior Engineering Specialist, Philco-Ford Corp. Member AIAA.

Comments on Precursors Ahead of Pressure Driven Shock Waves

HELMUT D. WEYMANN*
University of Rochester, Rochester, New York

Nomenclature

h = Planck's constant
k = Boltzmann's constant
k_ν = absorption coefficient
n = electron or ion number density
t = time
u_s = shock velocity
x = distance ahead of shock front
z = distance between measuring station and diaphragm station
z_c = minimum of z for which a stationary precursor profile can be observed
N = photon flux density, $N_0 = N$ at $x = 0$
T = absolute temperature
α = absorption coefficient
ν = frequency

Received December 12, 1966.
* Professor of Mechanical and Aerospace Sciences.

RECENTLY, W. G. Zinman[1] has discussed a possible explanation of the plasma density profiles ahead of pressure driven shock waves. Zinman based his theory on microwave measurements of Lederman and Wilson,[2] who found that the electron density profiles do not depend on the driven gas pressure and are stationary in a coordinate system moving with the shock front. These results are contrary to experimental findings of L. B. Holmes,[3] who used a collecting probe. In extensive investigations of the electron and ion densities in the precursor region, Holmes found that for Mach numbers ranging from 9 to 10.6, precursors propagating into argon at pressures between 2.5 and 10 torr do not become stationary in a shock fixed coordinate system up to diaphragm-probe distances of 310 cm. In addition, Holmes found that the plasma density at a given distance ahead of the shock front depends strongly on the driven gas pressure.

To explain the fact that their electron density profiles are independent of the gas pressure, Lederman and Wilson assumed that the atoms that are ionized in the precursor region are oxygen impurities whose partial pressure is independent of the gas pressure. Zinman, however, assumes that argon atoms are involved in at least the first step of excitation to a resonance level. He further assumes that the e-folding length of the precursor profile is inversely proportional to the distance ahead of the shock front. This is not born out by Lederman and Wilson's experiments.

Zinman's theoretical analysis is based on Wetzel's[4] treatment of photoionization in the precursor region. Unfortunately, Zinman makes several errors in going from his first to his second integral. These errors invalidate Zinman's formal derivation. There are, however, other assumptions that should be challenged. It would take too long to discuss all of Zinman's assumptions in detail. This however, is not necessary. His main point seems to be that he wants to show that $k_{\nu_{\rm eff}} \approx 1/x$ and that this makes the electron density profiles independent of the pressure. In the last paragraph of the first column Zinman states that the k_ν range of interest (for the second integral) is $k_\nu \approx 1/x$. This expression is later pulled out of the calculation as proven. In addition to the fact that Zinman has not proven that the range around $k_\nu \approx 1/x$ gives the major contribution to the integral, even his a priori assumption that $k_\nu \approx 1/x$ cannot be made for arbitrary frequency dependences of the radiant flux and the absorption coefficient. For the case of one-step photoionization, one can show that the major contribution to Wetzel's Eq. (19) comes from the range around

$$k_\nu \approx (5/x^{3/4})(3k_\nu{}^0kT/h\nu_i)^{1/4}$$

For $h\nu_i/kT = 15$ and $k_\nu{}^0 = 5\ 10^4\ \text{cm}^{-1}$, this expression becomes

$$k_\nu \approx 50/x^{3/4}$$

As k_ν is proportional to the density of absorbers, a change in the driven gas density will change the range of ν which, for a given x, will give the major contribution to Wetzel's Eq. (19). The magnitude of this contribution will change with the pressure because the photon flux density is a function of ν. For this reason there should be a pressure dependence of the plasma density profiles.

Finally, a comment will be made on the time necessary to establish a stationary plasma density profile relative to the shock front. As the driven gas is initially un-ionized, there must be a transient during which the stationary distribution is established. To find a lower limit for the distance which the shock has to travel until the plasma density at a distance x ahead of it has reached a constant value, let us assume that the photon flux density of the ionizing radiation is given by

$$N = N_0 \exp(-\alpha x)$$